Real Options and Private Investment Decisions

Carol Alexander and Xi Chen

ICMA Centre, Henley Business School

University of Reading

November 2011

Real Options *vs* Investment Decisions

- Myers (1977): A real option is a decision opportunity for a corporation or an individual whose value is contingent on the price of some underlying asset
- Research focuses on fixed strike American call pricing, continuous time, risk-neutral price for tradable contract
- Assumptions inappropriate for many real investment decisions – e.g. real estate, R&D, M&A
- Our aim: general framework for valuing all types of investment opportunities – real options just a special case

Risk-Neutral Pricing: Continuous Time

Fixed/Pre-determined Strike	Stochastic Strike
Triantis and Hodder (1990)	McDonald and Siegel (1986)
Capozza and Sick (1991)	Quigg (1993)
Kogut (1991)	Bowman and Moskowitz (2001)
Trigeorgis (1993)	
Grenadier (1996)	
Panayi and Trigeorgis (1998)	
Smith and McCardle (1998)	
Benaroch and Kauffman (2000)	
Boer (2000)	
Yeo and Qiu (2002)	

Note: Papers based on infinite horizon in blue

Risk-Neutral Pricing: Discrete Time

All papers use pre-determined strike and finite horizon:

Smit and Ankum (1993)
Smit (1996)
Trigeorgis (1996)
Smith and McCardle (1999)
Copeland and Antikarov (2001)
Copeland and Tufano (2004)
Brandão and Dyer (2005)
Brandão et al. (2005)
Smith (2005)
Brandão et al. (2008)

Valuation for Incomplete or Partially Complete Markets

Utility	Infinite Horizon	Finite Horizon
Exponential	Henderson (2007) Miao and Wang (2007)	Smith and Nau (1995) Henderson (2002) Grasselli (2011)
Power	Evans et al. (2008)	Henderson and Hobson (2002)
Logarithmic	Evans et al. (2008)	

Note: Only one paper in discrete time framework.

Some Open Questions

Questions addressed in the incomplete market context:

- How do the investor's subjective views about future market prices affect his valuation of an investment opportunity?
- For a given initial level of risk tolerance, how does the form of the investor's utility function affect the relative values of different opportunities?
- How does the frequency of decision opportunities affect their value to the investor?
- How does the size of an investment, relative to the investor's initial wealth, affect his valuation of an investment opportunity?

Mathematical Framework

- **Market**: Incomplete – investment risks un-hedgeable
- **Measure**: Subjective – RN measure as special case
- **Decision Opportunities**: Discrete – binomial price tree with decision nodes placed at every k steps
- **Valuation**: Utility – risk-averse and risk-neutral investors
- **Investment cost**: Flexible – may be pre-determined or, if stochastic, perfectly correlated with the market price

Market Prices

Market prices p_t of the investment are expressed in time 0 terms, discounting at the constant rate r. The process for p_t and its parameters are subjective:

- **GBM**:

$$\frac{dp_t}{p_t} = (\mu - r)dt + \sigma dW_t, \quad 0 < t < T$$

- **Boom-Bust**:

$$(\mu_1, \sigma_1) \text{ for } 0 < t < s \text{ and } (\mu_2, \sigma_2) \text{ for } s \leq t < T$$

- **Mean-reversion**:

$$d \ln p_t = -\kappa \ln\left(\frac{p_t}{\bar{p}}\right) dt + \sigma dW_t, \quad 0 < t < T$$

κ is the rate of mean reversion to long-term price level $\bar{p}$

Binomial Tree Parameterizations

GBM/BB: Jarrow and Rudd (1982)

$$u = e^{m+\sigma\sqrt{\Delta t}}, \quad d = e^{m-\sigma\sqrt{\Delta t}}, \quad m = [\mu - r - 0.5\sigma^2]\Delta t, \quad \pi = 0.5$$

Mean-Reversion: Nelson and Ramaswamy (1990)

$$u = e^{\sigma\sqrt{\Delta t}}, \quad d = u^{-1}$$

$$\pi_{\mathbf{s}(t)} = \begin{cases} 1, & 0.5 + \nu_{\mathbf{s}(t)}\frac{\sqrt{\Delta t}}{2\sigma} > 1 \\ 0.5 + \nu_{\mathbf{s}(t)}\frac{\sqrt{\Delta t}}{2\sigma}, & 0 \leq 0.5 + \nu_{\mathbf{s}(t)}\frac{\sqrt{\Delta t}}{2\sigma} \leq 1 \\ 0, & 0.5 + \nu_{\mathbf{s}(t)}\frac{\sqrt{\Delta t}}{2\sigma} < 0 \end{cases}$$

where $\nu_{\mathbf{s}(t)} = -\kappa \left(\ln p_{\mathbf{s}(t)u} - \ln \bar{p}\right)$ is the local drift in $\ln p_t$.

Cash Flows

Subjective and investment dependent

$$\mathrm{CF}_{\mathbf{s}(t)} = p^{+}_{\mathbf{s}(t)} - p^{-}_{\mathbf{s}(t)} \quad \text{and} \quad \delta_{\mathbf{s}(t)} = \frac{\mathrm{CF}_{\mathbf{s}(t)}}{p^{+}_{\mathbf{s}(t)}}$$

$\mathbf{s}(t)$: string of u's and d's with t elements
i.e. path of market price up to time t

$\mathrm{CF}_{\mathbf{s}(t)}$: cash flow when price is in state $\mathbf{s}(t)$

$p^{-}_{\mathbf{s}(t)}$: ex-div market price
excludes cash flows before and at time t

$p^{+}_{\mathbf{s}(t)}$: price right before $\mathrm{CF}_{\mathbf{s}(t)}$ is paid out

$\tilde{p}_{\mathbf{s}(t)}$: market price if no cash flows
(cum-div price if cash-flows re-invested)

Investor's Utility

Subjective and (arguably) investment *in*dependent

Hyperbolic absolute risk aversion (HARA) class:

$$U(w) = -\mathrm{sgn}\{1-\eta\}\left[1+\lambda^{-1}\eta\left(\frac{w-w_0}{w_0}\right)\right]^{1-\frac{1}{\eta}}$$

- w_0 = initial wealth, w = wealth at time t, 0
- λ = local relative risk tolerance at w_0
- η = sensitivity of local relative risk tolerance to w

Relative risk tolerance increases linearly with w at rate η

$\lambda \to \infty$ linear utility

Special One-Parameter Cases

Exponential $(\eta \to 0)$:

$$U(w) = -\exp\left(-\frac{w-w_0}{\lambda w_0}\right) \to -\exp\left(-\frac{w}{\lambda w_0}\right) \to -\exp\left(-\frac{w}{\lambda^*}\right)$$

Logarithmic $(\eta \to 1)$:

$$U(w) = \ln\left(1 + \frac{w-w_0}{\lambda w_0}\right), w > (1-\lambda)w_0$$

Power $(\eta = \lambda)$:

$$U(w) = -\text{sgn}\{1-\lambda\}\left(\frac{w}{w_0}\right)^{1-\frac{1}{\lambda}} \text{ and } w/w_0 > 0 \text{ for } \lambda < 1$$

Hyperbolic $(\eta = 0.5)$:

$$U(w) = -\left(1 + \frac{w-w_0}{2\lambda w_0}\right)^{-1} \text{ and } w/w_0 > 1-2\lambda \text{ for } U'(w) < 0$$

Cost of Investment

Investment cost at time t in time 0 terms:

$$K_{\mathbf{s}(t)} = \alpha K + (1-\alpha)p^{-}_{\mathbf{s}(t)}, \quad 0 \leq \alpha \leq 1$$

for K constant, i.e. fixed strike in time 0 terms

- $\alpha = 1 \rightarrow$ fixed cost, i.e. standard real option
  e.g. Oil exploration decisions
- $\alpha = 0.5 \rightarrow$ fixed cost + variable cost
  e.g. R&D decisions
- $\alpha = 0 \rightarrow$ variable cost, i.e. standard invest option
  e.g. Real estate and M&A decisions

We distinguish between *real option* with fixed strike and *invest option* where investment cost is at market price

Framework for Decision Analysis

Take the example of R&D in pharmaceutical products:

- Discrete decision opportunities: R&D committee meets periodically
- Assess *relative* values of R&D into different products
- Each project assessed at some fixed horizon, time T'
- Value is certainty equivalent (CE) of expected utility of terminal wealth, $w_{T'}$, minus initial wealth, w_0
- Choose project(s) with highest value and follow solution, i.e. commence R&D now or in future if market price has reached some given level

Initial and Terminal Wealth

Initial wealth w_0 not used to fund the investment $\rightarrow$ time 0 values computed using r, the investor's borrowing rate

Assume w_0 and cash flows earn constant lending rate $\tilde{r}$

$w^I_{t,T'}$ or $w^D_{t,T'}$ denotes total wealth at time T' following decision at time t, in time 0 terms (notation $\mathbf{s}(t) \rightarrow t$):

- If **I**nvest at time t

$$w^I_{t,T'} = e^{(\tilde{r}-r)T'} w_0 + \underbrace{\sum_{s=t+1}^{T'} e^{(\tilde{r}-r)(T'-s)} \mathrm{CF}_s + p^-_{T'}}_{\{\text{cum-div } \tilde{p}_{t,T'} \text{ if } \tilde{r}=r\}} - K_t$$

- If **D**efer at time t:

  $w^D_{t,T'}$ depends on whether he invests later on

Solution: Numerical Resolution

1. Sequence of optimal decisions at each decision node
   - Backward induction on expected utility:

$$\mathrm{E}\left[U_{\mathbf{s}(t),T'}\right] = \max\left\{\mathrm{E}\left[U\left(w^I_{\mathbf{s}(t),T'}\right)\right], \mathrm{E}\left[U\left(w^D_{\mathbf{s}(t),T'}\right)\right]\right\}$$

2. Value of investment opportunity is pay-off: $CE_0 - w_0$
   - CE_0 = certain equivalent of $\mathrm{E}[U_{\mathbf{s}(0),T'}]$, i.e.

$$U(CE_0) = \mathrm{E}[U_{\mathbf{s}(0),T'}]$$

Typical behaviour and values for $\alpha = 0$ are quite different to case $\alpha = 1$

Illustration of Solution

- Investment cost at market price
- GBM with $p_0 = 1$, $r = 5\%$, $\mu = 10\%$, $\sigma = 20\%$
- Exponential utility with $\lambda = 0.4$
- Decision opportunities every year for three years

Figure 1 in Appendix $\rightarrow$ decision tree with discounted market prices (black), expected utilities (blue), costs and P&L (red), and solution marked in bold

- Optimal decision at time 0 is to defer and invest only if market price falls next year, with EU $= -0.394$
- $\text{CE} = -0.4 \log(0.394/0.4) = 5,853 \times 10^{-6}$
- i.e. if $p_0 = \$1\text{m}$, the value of this decision opportunity to this investor is $\$5,853$, net of financing cost

Real Option *vs* Option to Invest at Market Price

$\alpha = 0 \rightarrow$ invest at market price: tend to invest if price falls

$\alpha = 1 \rightarrow$ fixed strike at \$1m: tend to invest if price rises

Table: Exponential utility with parameters (as previous example):
$T' = 3$ years, $\Delta t = k = 1$, $K = p_0 = \$1\text{m}$, $r = 5\%$, $\mu = 10\%$, $\sigma = 20\%$.

α \ λ	0.2	0.4	0.6	0.8	1	∞
0	0	**5,853**	45,305	70,656	86,976	161,374
50%	31,597	64,814	84,749	96,762	106,544	161,374
1	59,825	116,939	145,009	161,573	172,462	224,333

Value increases with risk tolerance

More Frequent Decisions → Value Increases

$T' = 5\text{yrs}$, $\Delta t = 1/12$, $T = T' - k\Delta t$, $K = p_0 = \$1\text{m}$, $r = 5\%$, $\mu = 15\%$, $\sigma = 50\%$

(a) $\alpha = 0$: invest at market price

k \ λ	0.2	0.4	0.6	0.8	1	∞
12	109.5	1,132	3,712	8,197	14,480	645,167
6	142.2	1,344	4,309	9,212	16,089	645,167
3	163.9	1,471	4,618	9,803	17,022	645,167
1	176.9	1,553	4,828	10,185	17,624	645,167

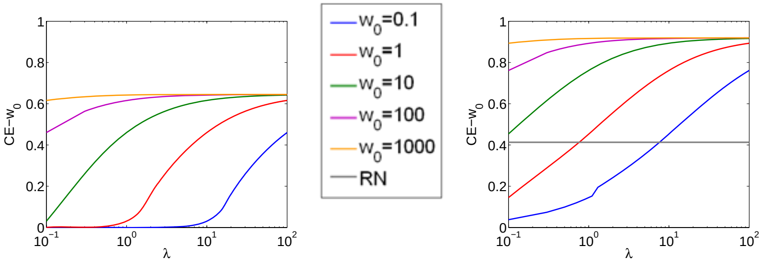
(b) $\alpha = 1$: strike at \$1m

k \ λ	0.2	0.4	0.6	0.8	1	∞
12	49,385	115,086	174,731	223,201	263,614	881,419
6	71,365	144,638	204,078	252,243	292,632	908,333
3	86,062	157,289	214,375	263,019	303,834	919,322
1	93,115	166,450	225,568	273,619	313,888	926,058

Value ↑ as Cost Relative to w_0 ↓

Logarithmic utility option values $(CE - w_0)$ for different w_0

(a) $\alpha = 0$ (b) $\alpha = 1$



Note: $T' = 5$, $\Delta t = 1/12$, $k = 3$, $K = p_0 = \$1\text{m}$, $r = 5\%$, $\mu = 15\%$, $\sigma = 50\%$, $w_0 = \$0.1, 1, 10, 100, 1000\text{m}$, $0.1 \le \lambda \le 100$, CE in \$m. Axis λ is in log scale.

Comparison with Risk-Neutral (RN) Valuation

- RN investor: Linear utility $\rightarrow$ decision opportunities valued at expected pay-off only
- RN measure: $\mu = r$ $\rightarrow$ Forward price is martingale

  (Recall everything expressed in time 0 terms)

$\alpha = 0$: Investment cost at market price

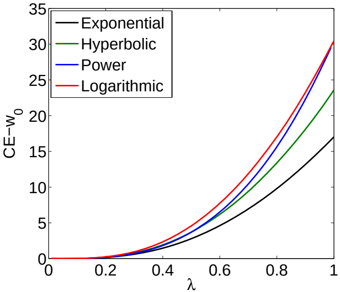
- Expected pay-off always zero $\rightarrow$ invest option value $= 0$

$\alpha = 1$: Fixed investment cost

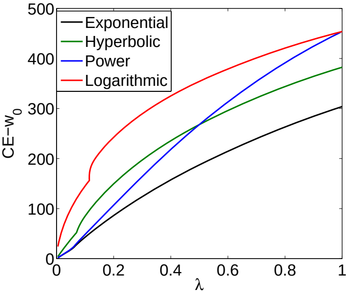
- Real option value = expected value of call option pay-off

Option Values ↑ as Risk Tolerance ↑

(a) $\alpha = 0$

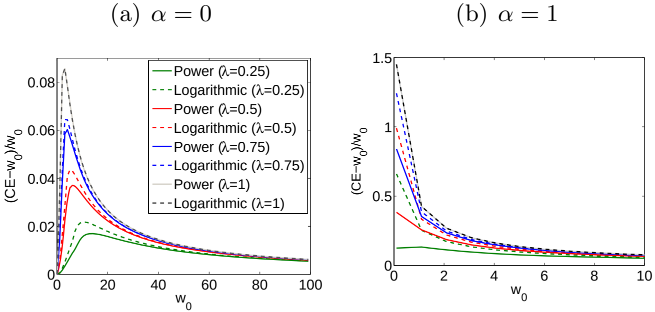


(b) $\alpha = 1$



Note: $T' = 5$, $\Delta t = 1/12$, $k = 3$, $K = p_0 = \$1\text{m}$, $r = 5\%$, $\mu = 15\%$, $\sigma = 50\%$, $w_0 = \$1\text{m}$, $0 < \lambda < 1$, CE in 000\$

Relative Value → Optimal Size



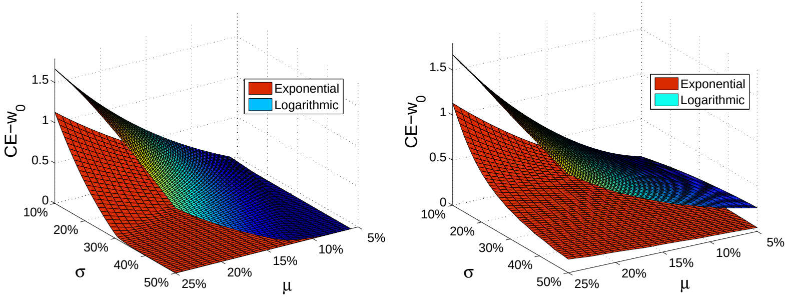
Note: $T' = 5$, $\Delta t = 1/12$, $k = 3$, $K = p_0 = \$1\text{m}$, $r = 5\%$, $\mu = 15\%$, $\sigma = 50\%$

- Smallest relative value is for large w_0, i.e. value for small projects does not depend on risk preference
- Optimal project size for $\alpha = 0$ and $\lambda = 0.5$ is about 1/5th of current wealth
- For $\alpha = 1$ the very large projects have greatest relative value

Sensitivity of Value w.r.t. μ and σ

$$\lambda = 0.2$$

(a) $\alpha = 0$ (b) $\alpha = 1$

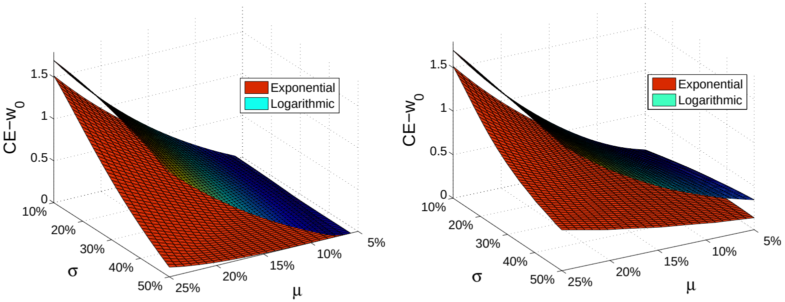


Note: $T' = 5$, $\Delta t = 1/12$, $k = 3$, $K = p_0 = \$1\text{m}$, $w_0 = \$10\text{m}$, $r = 5\%$

Sensitivity of Value w.r.t. μ and σ (cont.)

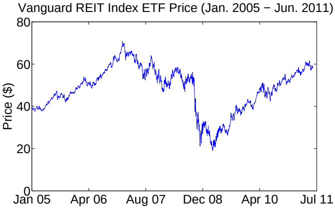
$$\lambda = 0.8$$

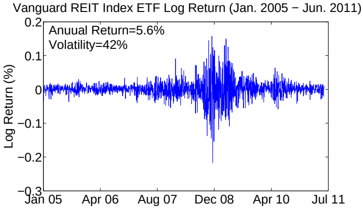
(a) $\alpha = 0$ (b) $\alpha = 1$



Note: $T' = 5$, $\Delta t = 1/12$, $k = 3$, $K = p_0 = \$1\text{m}$, $w_0 = \$10\text{m}$, $r = 5\%$

Scenario I – Property Price Bust and Boom





	Annual Return	Volatility
Jan. 05 - Dec. 06	21.25%	15%
Jan. 07 - Dec. 09	−13.07%	58%
Jan. 10 - Jun. 11	22.31%	24%

Example

Suppose investor believes the market price will **fall during the first n $(0 \leq n \leq 10)$ years** with

$$\begin{cases} \mu_1 = -10\% \\ \sigma_1 = 50\% \end{cases}$$

and then boom for the next $(10 - n)$ years with

$$\begin{cases} \mu_1 = 10\% \\ \sigma_1 = 20\% \end{cases}$$

Compare invest option values for different n

Parameters:

$$T' = 10,\ \Delta t = 1/4,\ k = 1,\ K = p_0 = w_0 = \$1\text{m},\ r = 5\%$$

Results – Exponential Utility

$\alpha = 0$

λ \ n	0	2	4	6	8	10
0.2	866	382,918	**727,922**	438,482	174,637	0
0.8	211,457	1,230,035	**2,297,909**	1,148,695	370,354	0
1.0	266,752	1,349,295	**2,552,434**	1,300,724	406,567	0
∞	648,173	2,234,361	4,777,950	**5,308,672**	1,045,800	0

$\alpha = 1$

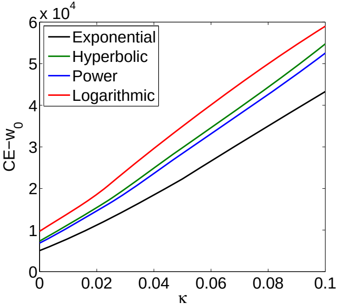
λ \ n	0	2	4	6	8	10
0.2	164,721	391,060	**552,578**	194,371	51,040	5,022
0.8	392,866	961,279	**1,704,007**	692,529	159,663	14,943
1.0	432,280	1,064,416	**1,938,157**	821,733	186,257	17,191
∞	740,493	1,989,551	4,339,659	**4,860,376**	841,706	50,787

Scenario II – Mean-Reverting Market Prices

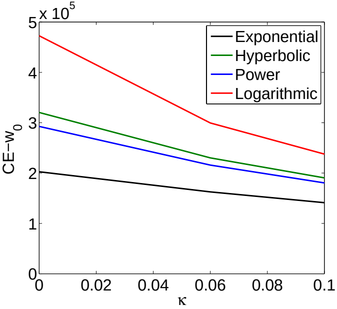
Characteristic time to mean-revert $\phi = \Delta t/\kappa$ in years

e.g. $\Delta t = 1/4$: $\kappa = 0.02 \rightarrow \phi = 12.5$yrs, $\kappa = 0.1 \rightarrow \phi = 2.5$yrs

(a) $\alpha = 0$



(b) $\alpha = 1$



Note: $w_0 = \$1$m, $r = 5\%$, $k = 1$ and $\lambda = 0.4$
$T' = 10$, $\Delta t = 1/4$, $K = p_0 = \$1$m, $T = T' - \Delta t$, $\sigma = 40\%$

Ranking Depends on Utility

Compare two potential investments:

$$\text{A: } \sigma = 20\%,\ \phi = 2.5\text{yrs}$$

View on property A price: low volatility, rapid mean reversion

$$\text{B: } \sigma = 40\%,\ \phi = 10\text{yrs}$$

View on property B price: high volatility, slow mean reversion

Different investors would rank them differently:

λ	0.4		0.8		∞	
	A	B	A	B	A	B
Exponential	**30,421**	12,952	**52,732**	43,947	98,077	**797,486**
Hyperbolic	**33,526**	17,563	55,230	**58,909**		
Power	**33,045**	16,651	56,365	**68,051**		
Logarithmic	19,890	**21,260**	56,986	**73,131**		

Real Estate Applications

Figures in appendix illustrate decision trees for:

1. Selling a property: divest option at market price $p^{+}_{\mathbf{s}(t)}$
2. Buy-to-let options: $\delta_t > 0$
3. Buy-to-develop options: $\delta_t < 0$

Buy-to-Let Options

- Cost of buying a rental property is $p^-_{\mathbf{s}(t)}$ and benefit of selling a rental property currently owned is $p^+_{\mathbf{s}(t)}$
- Decision trees depicted in figures 3 and 4 of appendix
- Relative value of two options depends on type of utility:

		Invest option		Divest option	
		A	B	A	B
Property / Utility	σ	40%	25%	30%	20%
	μ	15%	10%	15%	10%
	δ	10%	10%	20%	10%
Exponential		289	**316**	**6,775**	6,052
Hyperbolic		**348**	339	**6,610**	6,103
Power		340	**336**	**7,283**	6,296
Logarithmic		**358**	345	4,286	**5,330**

Note: $T' = 3$, $T = 2$, $k = 1$, $p_0 = w_0 = \$1\text{m}$, $\lambda = 0.4$, $r = 5\%$.

Buy-to-Develop Options

- Cost of land or property to develop is $p^{-}_{\mathbf{s}(t)}$
- Decision tree depicted in figure 5 of appendix
- Relative value of two options depends on type of utility:

Utility \ Property		A	B
	σ	25%	15%
	μ	10%	35%
	δ	-20%	-40%
Exponential		**289**	182
Hyperbolic		299	**326**
Power		315	**350**
Logarithmic		**41**	0

Note: If the land is acquired, the development takes 2 periods

$T = 2$, $k = 1$, $r = 5\%$, $\lambda = 0.4$, $p_0 = w_0 = \$1\text{m}$.

Select Conclusions

1. **Flexibility**: Option value $\uparrow$ as frequency of decision opportunities $\uparrow$
2. **Size**: Option value for risk-averse investor $\uparrow$ and $\rightarrow$ value for RN investor, as cost $\downarrow$ relative to investor's wealth
3. **Fixed Cost**: assumption often inappropriate – totally different optimal investment path, relative to variable costs assumption $\rightarrow$ misleading option value
4. **RN price**: also misleading – ignores both risk aversion and subjective views. Investors with:
    - different utilities but same views
    - different views but same utilities

    may rank the same investment decisions differently

References

M. Benaroch and R. J. Kauffman. Justifying electronic banking network expansion using real options analysis. *MIS Quarterly*, 24: 197–225, 2000.

P. F. Boer. Valuation of technology using "real options". *Research Technology Management*, 43:26–30, 2000.

E. H. Bowman and G. T. Moskowitz. Real options analysis and strategic decision making. *Organization Science*, 12:772–777, 2001.

L. E. Brandão and J. S. Dyer. Decision analysis and real options: A discrete time approach to real option valuation. *Annals of Operations Research*, 135(1):21–39, 2005. doi: 10.1007/s10479-005-6233-9.

L. E. Brandão, J. S. Dyer, and W. J. Hahn. Using binomial decision trees to solve real-option valuation problems. *Decision Analysis*, 2: 69–88, 2005.

References (cont.)

L. E. Brandão, J. S. Dyer, and W. J. Hahn. Response to comments on brandão et al. (2005). *Decision Analysis*, 2:103–109, 2008.

D. R. Capozza and G. A. Sick. Valuing long-term leases: the option to redevelop. *Journal of Real Estate Economics and Finance*, 4: 209–233, 1991.

T. Copeland and V. Antikarov. *Real Options: A Practitioner's Guide*. Texere, New York, 2001.

T. Copeland and P. Tufano. A real-world way to manage real options. *Harvard Business Review*, 1:1–12, 2004.

J. Evans, V. Henderson, and D. G. Hobson. Optimal timing for an indivisible asset sale. *Mathematical Finance*, 18(4):545–567, 2008.

M. R. Grasselli. Getting real with real options: A utility-based approach for finite-time investment in incomplete markets. *Journal of Business Finance and Accounting*, 38(5-6):740–764, 2011.

References (cont.)

S. R. Grenadier. The strategic exercise of options: development cascades and overbuilding in real estate markets. *The Journal of Finance*, 51:1653–1679, 1996.

V. Henderson. Valuation of claims on nontraded assets using utility maximization. *Mathematical Finance*, 12(4):351–373, 2002.

V. Henderson. Valuing the option to invest in an incomplete market. *Mathematics and Financial Economics*, 1(2):103–128, 2007.

R. Jarrow and A. Rudd. Approximate option valuation for arbitrary stochastic processes. *Journal of Financial Economics*, 10:347–369, 1982.

B. Kogut. Joint ventures and the option to expand and acquire. *Management Science*, 37:19–33, 1991.

R. McDonald and D. Siegel. The value of waiting to invest. *Quarterly Journal of Economics*, 101(4):707–727, 1986.

References (cont.)

J. Miao and N. Wang. Investment, consumption, and hedging under incomplete markets. *Journal of Financial Economics*, 86(3): 608–642, 2007.

S. C. Myers. Determinantes of corporate borrowing. *Journal of Financial Economics*, 5(2):147–175, 1977.

D. B. Nelson and K. Ramaswamy. Simple binomial processes as diffusion approximations in financial models. *The Review of Financial Studies*, 3:393–430, 1990.

S. Panayi and L. Trigeorgis. Multi-stage real options: the cases of information technology infrastructure and international bank expansion. *The Quarterly Review of Economics and Finance*, 38: 675–682, 1998.

L. Quigg. Empirical testing of real option-pricing models. *The Journal of Finance*, 48:621–640, 1993.

References (cont.)

H. T. J. Smit. The valuation of offshore concessions in the netherlands. *Financial Management*, 26(2):5–17, 1996.

H. T. J. Smit and L. A. Ankum. A real option and game-theoretic approach to corporate investment strategy under competition. *Financial Management*, 22(3):241–250, 1993.

J. E. Smith. Alternative approaches for solving real-options problems. *Deicison Analysis*, 2(2):89–102, 2005.

J. E. Smith and K. F. McCardle. Valuing oil properties: Integrating option pricing and decision analysis approaches. *Operations Research*, 46(2):198–217, 1998.

J. E. Smith and K. F. McCardle. Options in the real world: Lessons learned in evaluating oil and gas investments. *Operations Research*, 47:1–15, 1999.

References (cont.)

J. E. Smith and R. F. Nau. Valuing risky projects - option pricing theory and decision-analysis. *Management Science*, 41(5):795–816, 1995.

A. J. Triantis and J. E. Hodder. Valuing flexibility as a complex option. *The Journal of Finance*, 45:549–565, 1990.

L. Trigeorgis. The nature of option interactions and the valuation of investments with multiple real options. *The Journal of Financial and Quantitative Analysis*, 28:1–20, 1993.

L. Trigeorgis. *Real Options: Managerial Flexibility and Strategy in Resource Allocation.* The MIT Press, Cambridge, MA., 1996.

K. T. Yeo and F. Qiu. The value of management flexibility - a real option approach to investment evaluation. *International Journal of Project Management*, 21:243–250, 2002.