# Valuation ratios and shape predictability in the distribution of stock returns

Paolo Giordani and Michael Halling

Sveriges Riksbank

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# Nobel talk (2014) on predictability, bubbles, excess volatility

- Fama "... *stocks returns are somewhat predictable from dividend yields and interest rates, but ... there is no reliable evidence that expected returns are sometimes negative.... Thus ... confident statements about "bubbles" ... are based on beliefs, not reliable evidence."*
- Below mean perhaps, but not negative.
- Shiller "*Low R squared .. tell us only the obvious, and do not tell us about the rationality of markets*". Can Fama explain excess volatility?
- They both make statements (implicitly) about linear models and symmetric distributions (expected returns, R squared, variance, ...).

# What do traders say?

- *"Up the stairs, down the elevator."* Trader's adage.
- *"Up the escalator, down the chute."* Trader's adage.
- *"Crashes follow distorsions." (Excessive valuations)* Mark Spietznagel, hedge fund manager.

No symmetry or linearity here.

# Our project

- Perhaps we can learn more about predictability and the role of valuation ratios if we do not force symmetry.
- What's a potentially fruitful way of modelling asymmetry in the predictive distribution of stock returns?
- How about extending the standard regression to allow valuation ratios to affect not only the mean of the predictive distribution, but the shape as well?
- That sounds reasonable, but please also provide a theoretical motivation.

# A simple model of "up the escalator, down the chute" (Blanchard and Watson, 1982)

Blanchard and Watson's model of rational bubbles

$$
\begin{aligned}
p_t &= p_t^* + c_t \\
E(c_{t+1}|\Omega_t) &= (1+r)c_t,
\end{aligned}
$$

call $c_t$ the "valuation ratio".
Simple deterministic bubble ("*implausible*")

$$c_t = (1+r)c_{t-1}.$$

Simple stochastic bubble

$$c_t = (1+r)c_{t-1}(1-\pi)^{-1}, \text{ with prob. } 1-\pi$$

$$c_t = \mu_t \text{ with prob } \pi,$$

$$\text{where } E(\mu_t|\Omega_{t-1}) = 0.$$

# Time varying asymmetry in the Blanchard and Watson model

- Blanchard and Watson focus the empirical work on excess volatility and kurtosis.
- Assume that changes in the the fundamental price $p_t^*$ have a symmetric distribution. Returns are a mixture of two distributions. At $c_t = 0$, the one step ahead distribution is symmetric. At $c_t > 0$, the distribution is increasingly left skewed.
- *The shape of the predictive distribution depends on the valuation ratio.*

# The Blanchard Watson model and predictability

- Campbell and Shiller (see also Cochrane): valuation ratios must either predict future dividends/earnings (they don't) or future returns.
- Yet in the BW model they don't. What's happening?
- Campbell and Shiller: *"If we accept the premise that valuation ratios will continue to fluctuate within their historical ranges in the future ...* " Technically, valuation ratios are assumed stationary.
- In the BW model, the valuation ratio may not fluctuate within any "historical" range, no matter how long the history.
- We don't need to go all the way to the BW world and assume constant expected returns.
- **Our take-away: valuation ratios may carry information on the shape of the distribution, not just the mean.**

# What's in the literature?

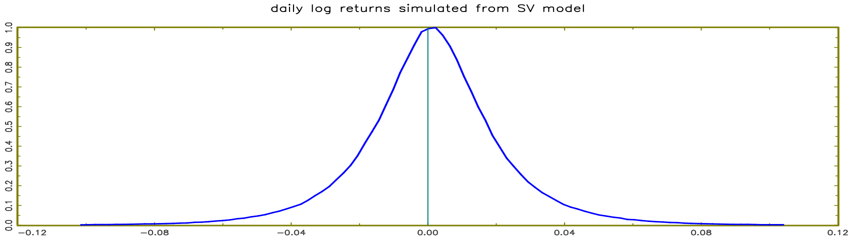
- Harvey and Siddique, "*Autoregressive Conditional Skewness*".
  - Skew-t for daily returns, where the skew parameter is a GARCH(2,2).
  - No relation to valuation ratios.
- Chen, Hong, and Stein, "*Forecasting crashes: trading volume, past returns, and conditional skeweness in stock prices*".
  - Model skewenss in a cross section of daily returns (six-months periods), regress it on volume, recent returns, and valuation ratios

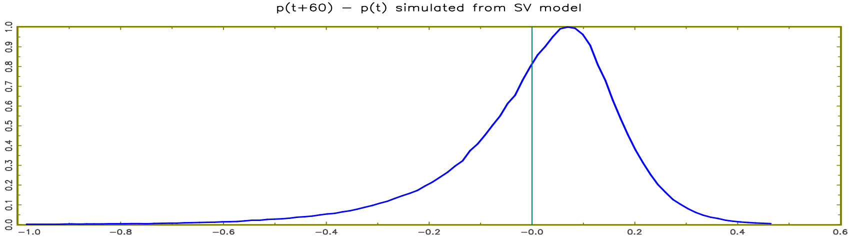
$$SKEW_t = \frac{\sum R_{t-h}^3}{(\sum R_{t-h}^2)^{3/2}}.$$

# Skeweness in daily returns is not skeweness in cumulative returns

- We are interested in asymmetry in the forecast distribution of cumulative returns at horizons of several months or years.
- Skeweness in daily returns may have little or no connection to time varying asymmetric shapes of cumulative returns. For example:
    1. Skewed daily returns can quickly converge to roughly symmetric cumulative returns.
    2. Symmetric daily returns can generate highly asymmetric cumulative (eg monthly or quarterly) returns.

# Example: symmetric daily returns and asymmetric cumulative returns from a SV model with leverage



# Our statistical model

We model what we are interested in directly, with a minimum of additional assumptions, staying as close as possible to well-known predictive regressions (in fact, nesting them).
Cumulative returns follow a skew-t distribution:

$$y_{t,t+h} \sim skewt(m_t, \sigma_t, v, \gamma_t),$$

where *skewt* is the skew-t distribution of Fernandez and Steel (JASA 1998), and $y_{t,t+h} = p_{t+h} - p_t$.
The model parameters are deterministic functions of covariates:

$$\begin{aligned} m_t &= \beta_{0,m} + \beta_{1,m} x_t \\ \ln \sigma_t &= \beta_{0,\sigma} + \beta_{1,\sigma} x_t \\ \ln v &= \beta_{0,v} \\ \ln \gamma_t &= \beta_{0,\gamma} + \beta_{1,\gamma} x_t. \end{aligned}$$

# On "skeweness", "shape", and "asymmetry".

The shape/asymmetry parameter $\gamma$ controls the allocation of mass to each side of the mode

$$\frac{P(y \geq m|\gamma)}{P(y < m|\gamma)} = \gamma^2.$$

For unimodel distributions, Arnold and Groeneveld (1995) propose a measure of skeweness defined as one minus twice the probability mass left of the mode, which in our case is

$$\frac{\gamma^2 - 1}{\gamma^2 + 1},$$

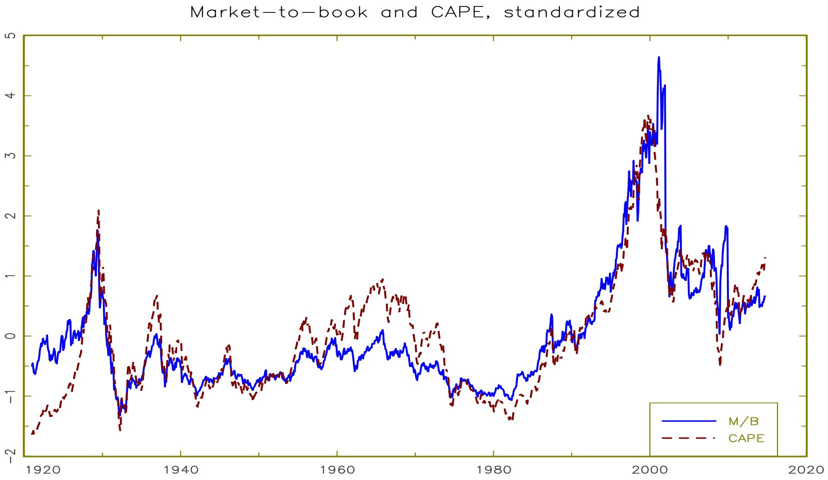
so symmetry is $\ln \gamma_t = 0$. Skeweness defined as $m_3/\sqrt{m_2}$ is a function of $\sigma_t, v, \gamma_t$.

# Estimation

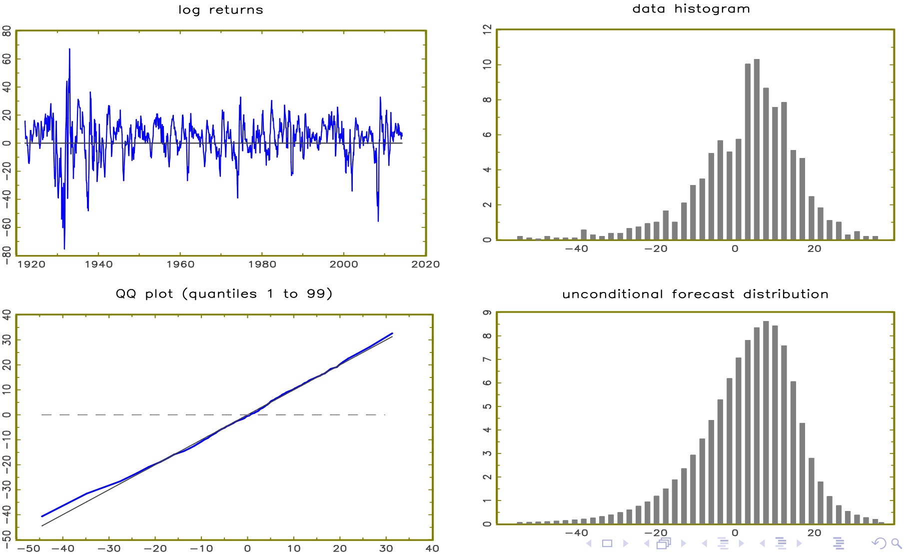
- ML possible, if a bit tricky.
- We opt for Bayesian inference using a MCMC approach
- We follow Villani, Giordani and Kohn, JoE, and Villani et al., JoE.
- Slowish but reliable even for very complex models.
- (Optional) Adjustment for overlapping observations, inspired by frequentist autocorrelation-correction (but otherwise ad hoc), divide log-likelihood by $1 + (h - 1)/2$.
- Weak Gaussian priors and no variable selection. Prior on $\beta_{1,\gamma}$ centered at zero.
- Degrees of freedom prior truncated at 3.
- Parameter uncertainty integrated out.

# The data

Goyal and Welch, monthly data 1981-2014 (1921 for BM).



# Unconditional distribution fit (six months)

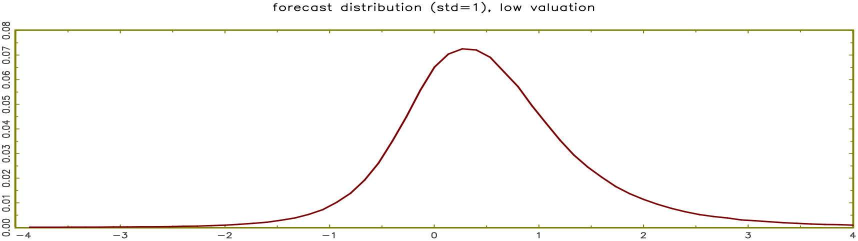


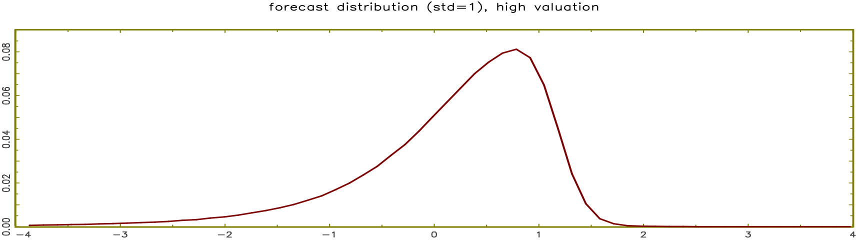
# Estimates of asymmetry parameters

Valuation ratio standardized (zero mean, unit std).

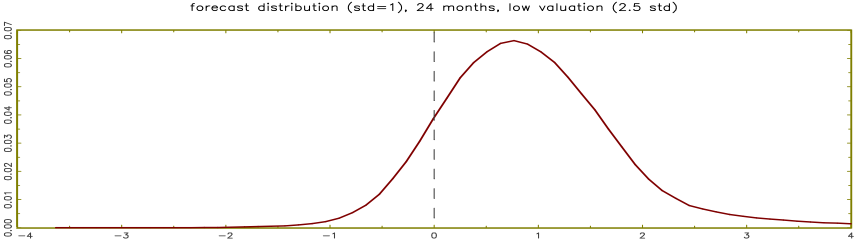
| | BM, 6 m | BM, 24 months |
|---|---|---|
| mean of $\beta_{0,\gamma}$ | $-0.37$ | $-0.47$ |
| mean of $\beta_{1,\gamma}$ | 0.25 | 0.36 |
| 5% of $\beta_{1,\gamma}$ | 0.10 | 0.05 |
| 95% of $\beta_{1,\gamma}$ | 0.40 | 0.65 |

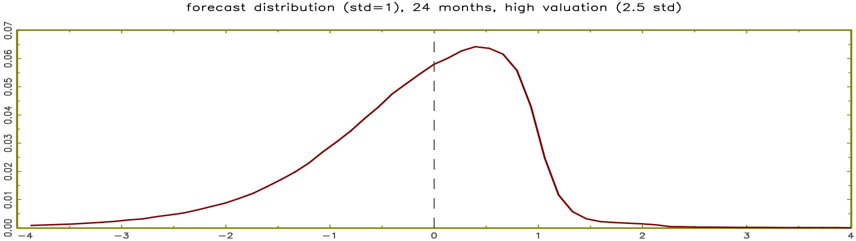
# At six months



# At two years


forecast distribution (std=1), 24 months, low valuation (2.5 std)
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
−4
−3
−2
−1
0
1
2
3
4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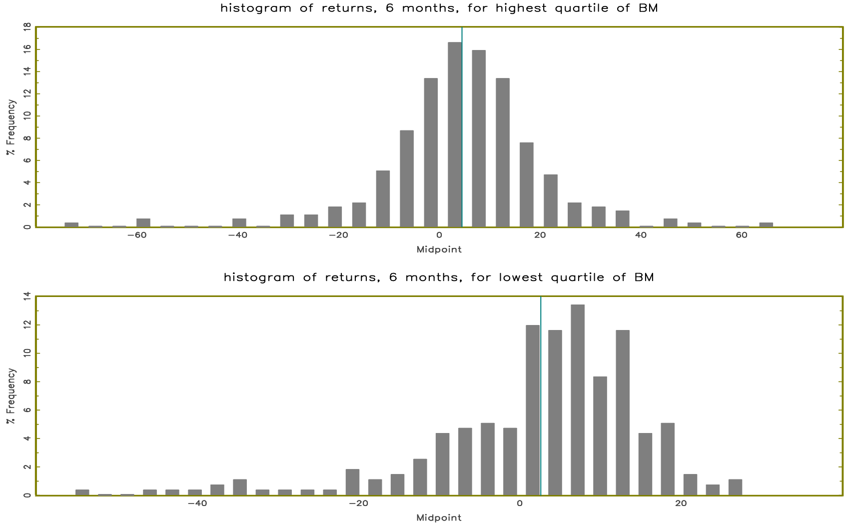
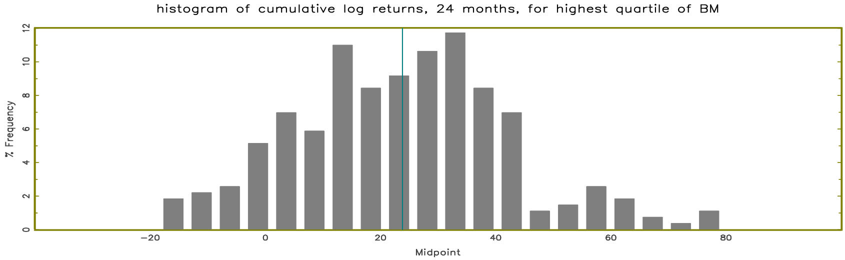
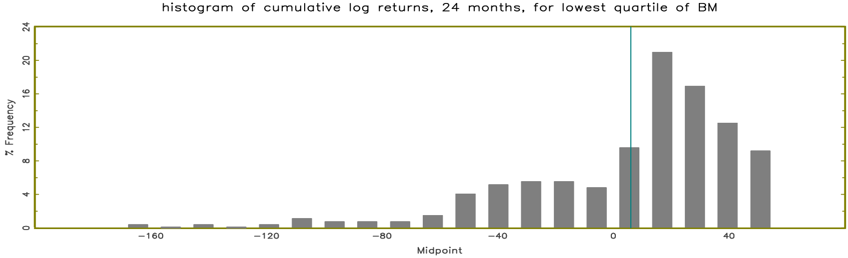
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
−4
−3
−2
−1
0
1
2
3
4

# And without using a model? At six months.

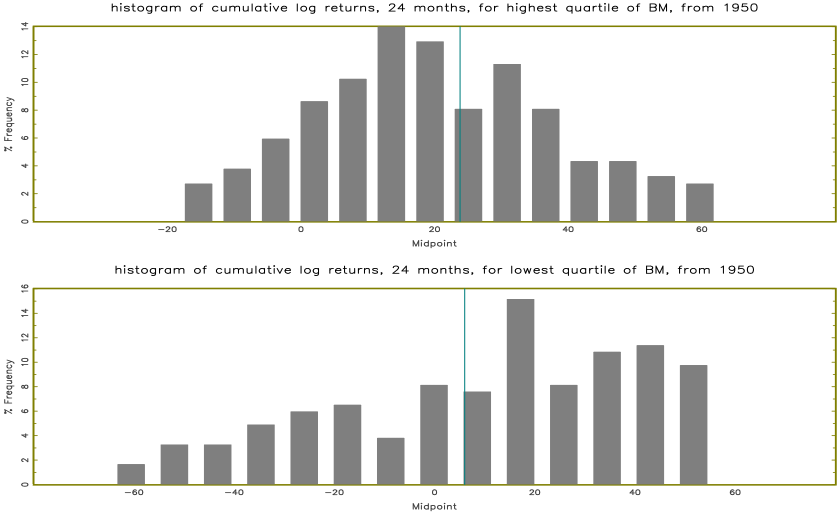


# At two years.

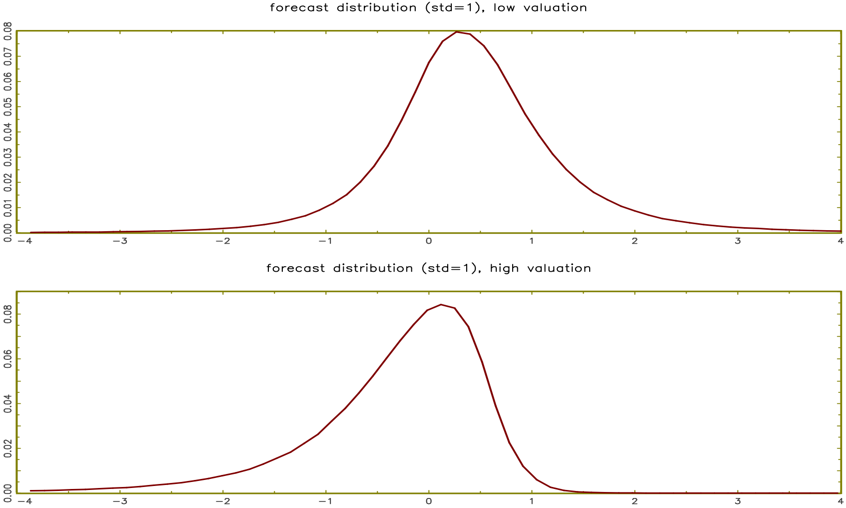

histogram of cumulative log returns, 24 months, for highest quartile of BM
% Frequency
Midpoint
−20
0
20
40
60
80
0
2
4
6
8
10
12


histogram of cumulative log returns, 24 months, for lowest quartile of BM
% Frequency
Midpoint
−160
−120
−80
−40
0
40
0
4
8
12
16
20
24

# At two years 1950-2014



# Credit spread (AAA-gov), 1960-2014, six months.



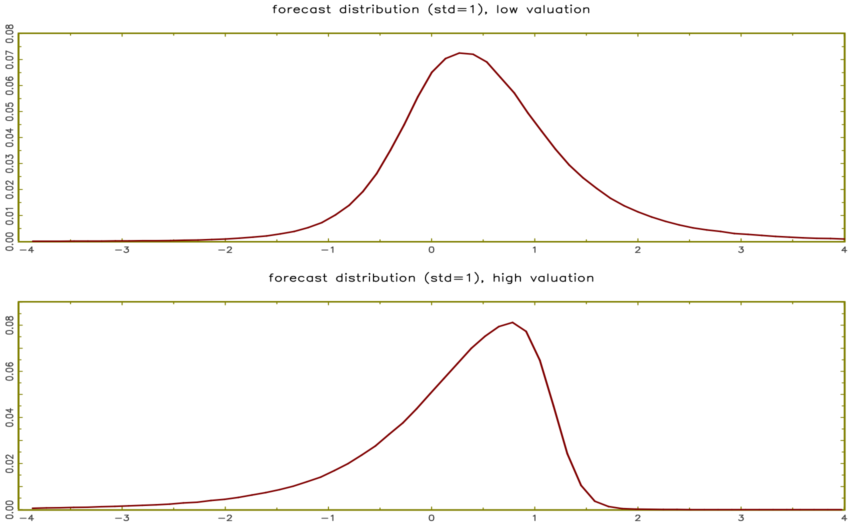
# Some implications

1. For a mean-variance investor, a standard model will underestimate (overestimate) the variance at high (low) valuations.
2. For a CRRA investor, the thinner (high BM) or thicker (low BM) left tail matters further.
3. Time varying asymmetry makes "timing" harder at high valuations. (More ahead)
4. Empirical findings of parameter instability in OLS regressions use linear and Gaussian models ... (More ahead)

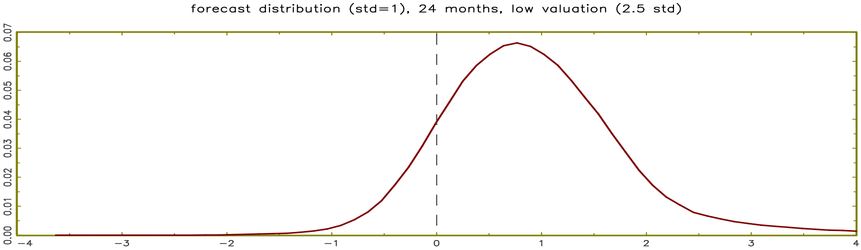
# Implications for "timing"

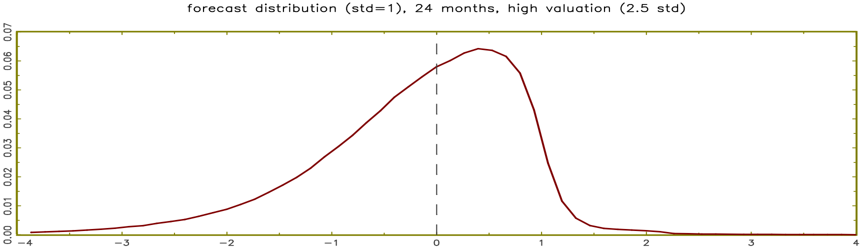
- Fama's Nobel talk: Shiller sounded the alarm in 1996 ...
- With a symmetric distribution, good "timing" (directional prediction) is possible starting from the tails (very high or low valuations).
- A highly asymmetric distribution (which keeps getting more asymmetric) makes timing harder than a symmetric distribution at high valuations.
- We find that the probability of a positive return in the next 12 months is well above 50% even at extremely high valuations.

# Easy and difficult timing. Six months.



# Easy and not so easy timing. Two years.



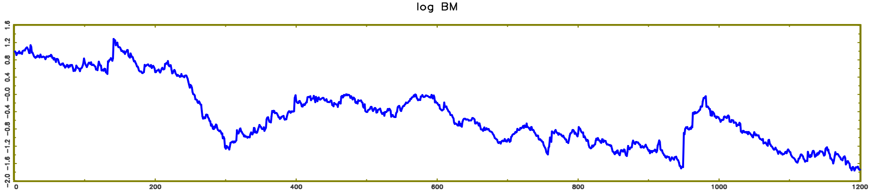



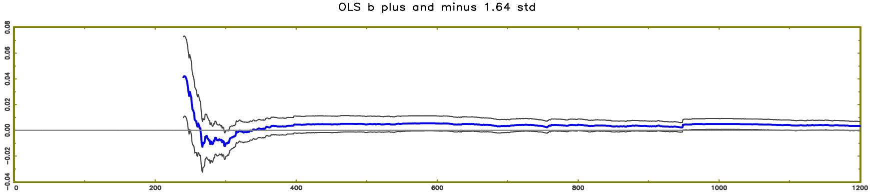
# Implications for parameter stability in regressions (conjectured).

From simulated data (ML estimates on 1 month):

- No (linear) predictability most likely found at highest valuations!
- BM ratio can wander for very long periods, even though it is stationary in this simulation.


log prices
12
10
8
6
4
2
0
200
400
600
800
1000
1200


log BM
1.6
1.2
0.8
0.4
-0.0
-0.4
-0.8
-1.2
-1.6
-2.0
0
200
400
600
800
1000
1200


OLS b plus and minus 1.64 std
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
0
200
400
600
800
1000
1200

# Possible further work

- Extend to more indexes. We started "easy". Think Nikkei, Nasdaq, ...
- Extend to cross-sections.
- Higher frequency statistical models (e.g. SV).
- Theoretical models: which can and cannot generate time-varying asymmetry?